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GENERALIZATIONS OF WEAKER FORMS OF OPEN MAPS IN FERMATEAN FUZZY TOPOLOGICAL SPACES AND THEIR APPLICATIONS

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Abstract: In this paper, we study Fermatean fuzzy Z-open and Fermatean fuzzy Z-closed functions within the framework of Fermatean fuzzy topological spaces. We investigate several significant properties of these functions and examine their relationships. In addition, we present a real-life decision-making application based on an entropy measure for Fermatean fuzzy sets, illustrating their practical usefulness in real-world situations.

Keywords and Phrases: Fermatean fuzzy Z-open mappings, Fermatean fuzzy Z-closed mappings, entropy measure.

2020 Mathematics Subject Classification: 03E72, 54A05, 54A40.

1. Introduction

Fuzzy sets were introduced by Zadeh [30] in 1965. The concept of fuzzy sets laid the foundation for the mathematical representation of vague and imprecise phenomena that occur in the real world. Since then, this notion has given rise to several new branches of mathematics and has found valuable applications in areas such as statistics, data processing, and linguistics. A large body of research has been devoted to fuzzy sets and their applications ever since their introduction.

In 1968, Chang [6] extended fuzzy set theory to topological spaces by generalizing fundamental topological concepts such as open sets, closed sets, continuity, and compactness to the framework of fuzzy topological spaces. Later, Atanassov [1] introduced intuitionistic fuzzy sets, and many further contributions were made by him and his collaborators [2, 5]. Coker [7] studied intuitionistic fuzzy topological spaces, while Yager [28] introduced a non-standard fuzzy set known as the Pythagorean fuzzy set. The study of Pythagorean fuzzy topological spaces was subsequently developed by Olgun et al. [13]. In related aggregation-based fuzzy decision frameworks, Rawat et al. [18] developed Archimedean t-norm based q-rung orthopair fuzzy Hamy mean operators together with ordinal priority weights, showing how algebraic structure and weighting mechanisms can be combined in multi-criteria decision-making. Similarly, Musbah and Badi [10] proposed an LLM-assisted virtual expert weight elicitation framework based on Z-numbers, illustrating a modern computational approach for handling subjective weights in uncertain decision environments.

In 2020, Senapati and Yager [17] proposed Fermatean fuzzy sets, which are capable of handling uncertain information more effectively in decision-making processes. These sets extend classical fuzzy sets by incorporating an additional parameter that increases their flexibility in representing uncertainty. Fermatean fuzzy sets are therefore well suited to more complex decision-making situations, and the basic operations on Fermatean fuzzy sets were defined in their work.

Hariwan Z. Ibrahim also introduced the concept of Fermatean fuzzy topological spaces and investigated the continuity of functions between such spaces. In this paper, we develop the notions of stronger and weaker forms of Fermatean fuzzy open sets in Fermatean fuzzy topological spaces and examine some of their basic properties through examples.

Research Gap. Despite extensive research on fuzzy sets and their generalizations, no substantial investigation has been carried out on the stronger and weaker forms of Fermatean fuzzy open maps. In particular, there is a lack of study on Fermatean fuzzy δ-open maps, Fermatean fuzzy δ-semi open maps, Fermatean fuzzy pre-open maps, Fermatean fuzzy Z-open maps, strongly Fermatean fuzzy Z-open maps, and perfectly Fermatean fuzzy Z-open maps in Fermatean fuzzy topological spaces. This reveals an important gap in the current literature.

The main objective of this paper is to address this gap by studying Fermatean fuzzy Z-open and Fermatean fuzzy Z-closed functions in Fermatean fuzzy topological spaces. We also introduce an entropy measure for Fermatean fuzzy sets and present an example to demonstrate its application in a real-life decision-making problem.

The proposed method has the advantage of providing a simple and effective way to measure fuzziness in Fermatean fuzzy sets, while also capturing the relationship between membership and non-membership information. This makes it suitable for practical decision-making problems, as illustrated by the application example. Another advantage is that the method is easy to compute and can help compare alternatives under uncertain and imprecise information.

At the same time, the method has some limitations. Its effectiveness depends on the quality and reliability of the subjective evaluations provided by decision-makers, and it may be less informative in highly complex real-world situations where additional criteria or interactions among parameters must be considered. Also, since the method is developed within the framework of Fermatean fuzzy sets, its direct applicability is limited to problems that can be modeled in this setting.

2. Preliminaries

We recall some basic notions of fuzzy sets, IFS's and pfs's.

Definition 2.1. [30] *Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function $\mu_A : X \to [0,1]$. That is:*

$$\mu_A(x) = \begin{cases} 1, & \text{if} \quad x \in X \\ 0, & \text{if} \quad x \notin X \\ (0,1) & \text{if } x \text{ is partly in } X. \end{cases}$$

Alternatively, a fuzzy set A in X is an object having the form $A = \{< x, \mu_A(x) > | x \in X\}$ or $A = \left\{ \left\langle \frac{\mu_A(x)}{x} \right\rangle | x \in X \right\}$, where the function $\mu_A(x) : X \to [0,1]$ defines the degree of membership of the element, $x \in X$.

The closer the membership value $\mu_A(x)$ to 1, the more x belongs to A, where the grades 1 and 0 represent full membership and full nonmembership. Fuzzy set is a collection of objects with graded membership, that is, having degree of membership. Fuzzy set is an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in a binary terms according to a bivalent condition; an element either belongs or does not belong to the set. Classical bivalent sets are in fuzzy set theory called crisp sets. Fuzzy sets are generalized classical sets, since the indicator function of classical sets is special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. Fuzzy sets theory permits the gradual assessment of the membership of element in a set; this is described with the aid of a membership function valued in the real unit interval $[0, 1]$.

Let us consider two examples:
(i) all employees of XYZ who are over $1.8m$ in height;
(ii) all employees of XYZ who are tall. The first example is a classical set with a universe (all XYZ employees) and a membership rule that divides the universe into members (those over $1.8m$) and nonmembers. The second example is a fuzzy set, because some employees are definitely in the set and some are definitely not in the set, but some are borderline.

This distinction between the ins, the outs, and the borderline is made more exact by the membership function, μ. If we return to our second example and let A represent the fuzzy set of all tall employees and x represent a member of the universe X (i.e. all employees), then $\mu_A(x)$ would be $\mu_A(x) = 1$ if x is definitely tall or $\mu_A(x) = 0$ if x is definitely not tall or $0 < \mu_A(x) < 1$ for borderline cases.

Definition 2.2. [1] *The intuitionistic fuzzy sets are defined on a non-empty sets X as objects having the form $I = \{\langle x, \mu_I(x), \lambda_I(x)\rangle : x \in X\}$, where $\mu_I(x) : X \to [0, 1]$ and $\lambda_I(x) : X \to [0, 1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set I, respectively, and $0 \leq \mu_I(x) + \lambda_I(x) \leq 1$, for all $x \in X$.*

Definition 2.3. [1, 2, 3, 4] *Let a nonempty set X be fixed. An IFS A in X is an object having the form:*

$A = \{< x, \mu_A(x), \lambda_A(x) > |x \in X\}$ or $A = \left\{\left\langle \frac{\mu_A(x), \lambda_A(x)}{x} \right\rangle |x \in X\right\}$, where the functions $\mu_A(x) : X \to [0, 1]$ and $\lambda_A(x) : X \to [0, 1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X : 0 \leq \mu_A(x) + \lambda_A(x) \leq 1$. For each A in X: $\pi_A(x) = 1 - \mu_A(x) - \lambda_A(x)$ is the intuitionistic fuzzy set index or hesitation margin of x in X. The hesitation margin $\pi_A(x)$ is the degree of nondeterminacy of $x \in X$ to the set A and $\pi_A(x) \in [0, 1]$. The hesitation margin is the function that expresses lack of knowledge of whether $x \in X$ or $x \notin X$. Thus: $\mu_A(x) + \lambda_A(x) + \pi_A(x) = 1$.

Example 2.1. Let $X = \{x, y, z\}$ be a fixed universe of discourse and $A = \left\{ \left\langle \frac{0.6,0.1}{x} \right\rangle, \left\langle \frac{0.8,0.1}{y} \right\rangle, \left\langle \frac{0.5,0.3}{z} \right\rangle \right\}$, be the intuitionistic fuzzy set in X. The hesitation margins of the elements x, y, z to A are as follows: $\pi_A(x) = 0.3$, $\pi_A(y) = 0.1$ and $\pi_A(z) = 0.2$.

Definition 2.4. [27, 28, 29] *Let X be a universal set. Then, a Pythagorean fuzzy set A, which is a set of ordered pairs over X, is defined by the following: $A = \{< x, \mu_A(x), \lambda_A(x) | x \in X\}$ or $A = \left\{ \left\langle \frac{\mu_A(x), \lambda_A(x)}{x} \right\rangle | x \in X \right\}$, where the functions $\mu_A(x) : X \to [0, 1]$ and $\lambda_A(x) : X \to [0, 1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X$, $0 \le (\mu_A(x))^2 + (\lambda_A(x))^2 \le 1$. Supposing $(\mu_A(x))^2 + (\lambda_A(x))^2 \le 1$, then there is a degree of indeterminacy of $x \in X$ to A defined by $\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\lambda_A(x))^2]}$ and $\pi_A(x) \in [0, 1]$. In what follows, $(\mu_A(x))^2 + (\lambda_A(x))^2 + (\pi_A(x))^2 = 1$. Otherwise, $\pi_A(x) = 0$ whenever $(\mu_A(x))^2 + (\lambda_A(x))^2 = 1$. We denote the set of all PFS's over X by $pfs(X)$.*

Definition 2.5. [17] *Let X be a universe of discourse. A Fermatean fuzzy set ($\mathfrak{F}\mathcal{F}s$) F in X is an object having the form $F = \{< x, \mu_F(x), \lambda_F(x) >: x \in X\}$ where $\mu_F(x) : X \to [0, 1]$ and $\lambda_F(x) : X \to [0, 1]$, including the condition $0 \le (\mu_F(x))^3 + (\lambda_F(x))^3 \le 1$, for all $x \in X$. The numbers $\mu_F(x)$ and $\lambda_F(x)$ denote, respectively, the degree of membership and the degree of non-membership of the element x in the set F. For any $\mathfrak{F}\mathcal{F}s$ F and $x \in X$, $\pi_F(x) = \sqrt[3]{1 - [(\mu_F(x))^3 - (\lambda_F(x))^3]}$ is identified as the degree of interminancy of x to F. In the interest of simplicity, we shall mention the symbol $F = (\mu_F, \lambda_F)$ for the $\mathfrak{F}\mathcal{F}s$ $F = \{< x, \mu_F(x), \lambda_F(x) : x \in X\}$.*

Definition 2.6. [17] *Let $F = (\mu_F, \lambda_F)$, $F_1 = (\mu_{F_1}, \lambda_{F_1})$ and $F_2 = (\mu_{F_2}, \lambda_{F_2})$, be three Fermatean fuzzy sets ($\mathfrak{F}\mathcal{F}s$'s), then their operations are defined as follows:*

(i) $F_1 \cap F_2 = (min\{\mu_{F_1}, \mu_{F_2}\}, max\{\lambda_{F_1}, \lambda_{F_2}\})$.

(ii) $F_1 \cup F_2 = (max\{\mu_{F_1}, \mu_{F_2}\}, min\{\lambda_{F_1}, \lambda_{F_2}\})$.

(iii) $F^c = (\lambda_F, \mu_F)$.

Remark 2.1. *If $\mu_{F_1} = \mu_{F_2}$ and $\lambda_{F_1} = \lambda_{F_2}$, then $F_1 = F_2$.*

Definition 2.7. [9] *Let X be a non empty set and τ be a family of Fermatean fuzzy subsets of X. If*

(i) $1_F, 0_F \in \tau$

(ii) *for any* $F_1, F_2 \in \tau$, *we have* $F_1 \cap F_2 \in \tau$,

(iii) *for any* $\{F_i\}_{i \in I} \subset \tau$, *we have* $\bigcup_{i \in I} F_i \in \tau$ *where* I *is an arbitrary index set then* τ *is called a Fermatean fuzzy topology on* X.
The pair (X, τ) *is said to be a Fermatean fuzzy topological space. Each member of* τ *is called an Fermatean fuzzy open set. The complement of an Fermatean fuzzy open set is called a Fermatean fuzzy closed set.*

Remark 2.2. [9] *As any Intuitionistic fuzzy subset or Pythagorean fuzzy subset of a set can be considered as Fermatean fuzzy subset, we observe that any Intuitionstic fuzzy topological space or Pythagorean fuzzy topological space is a Fermatean fuzzy topological space as well. On the other hand, it is obvious that a Fermatean fuzzy topological space need not be Intuitionistic fuzzy topological space and Pythagorean fuzzy topological space. Even an Fermatean fuzzy open set maybe neither an Intuitionistic fuzzy set nor Pythagorean fuzzy set.*

Example 2.2. [9] Let $X = \{c_1, c_2\}$. Consider the following family Fermatean fuzzy subsets $\tau = \{1_F, 0_F, F_1, F2\}$ where
$F_1 = \{\langle c_1, \mu_{F_1}(c_1) = 0.4, \lambda_{F_1}(c_1) = 0.6\rangle, \langle c_2, \mu_{F_1}(c_2) = 0.1, \lambda_{F_1}(c_2) = 0.3\rangle\}$ and $F_2 = \{\langle c_1, \mu_{F_2}(c_1) = 0.9, \lambda_{F_2}(c_1) = 0.6\rangle, \langle c_2, \mu_{F_2}(c_2) = 0.2, \lambda_{F_2}(c_2) = 0.3\rangle\}$. Observe that (X, τ) is a Fermatean fuzzy topological space but (X, τ) is neither Intuitionistic fuzzy topological space nor Pythagorean fuzzy topological space.

Definition 2.8. [9] *Let* (X, τ) *be an* $\mathfrak{F}\mathcal{F}ts$ *and* $A = \{< a, \mu_A(a), \lambda_A(a) > | a \in X\}$ *be an* $\mathfrak{F}\mathcal{F}s$ *in* X. *Then the Fermatean fuzzy interior and the Fermatean fuzzy closure of* A *are denoted by* $\mathfrak{F}\mathcal{F}int(A)$ *and* $\mathfrak{F}\mathcal{F}cl(A)$ *and are defined as follows:* $\mathfrak{F}\mathcal{F}int(A) = \cup\{G | G \text{ is a } \mathfrak{F}\mathcal{F}os \text{ and } G \subseteq A\}$ *and* $\mathfrak{F}\mathcal{F}cl(A) = \cap\{K | K \text{ is a } \mathfrak{F}\mathcal{F}cs \text{ and } A \subseteq K\}$. *Also, it can be established that* $\mathfrak{F}\mathcal{F}cl(A)$ *is an* $\mathfrak{F}\mathcal{F}cs$ *and* $\mathfrak{F}\mathcal{F}int(A)$ *is an* $\mathfrak{F}\mathcal{F}os$, A *is an* $\mathfrak{F}\mathcal{F}cs$ *if and only if* $\mathfrak{F}\mathcal{F}cl(A) = A$ *and* A *is an* $\mathfrak{F}\mathcal{F}os$ *if and only if* $\mathfrak{F}\mathcal{F}int(A) = A$. *We say that* A *is* $\mathfrak{F}\mathcal{F}$*-dense if* $\mathfrak{F}\mathcal{F}cl(A) = 1_{\mathfrak{F}}$.

Lemma 2.1. [9] *For any Fermatean fuzzy set* A *in* (X, τ), *we have* $1_{\mathfrak{F}} - \mathfrak{F}\mathcal{F}int(A) = \mathfrak{F}\mathcal{F}cl(1_{\mathfrak{F}} - A)$ *and* $1_{\mathfrak{F}} - \mathfrak{F}\mathcal{F}cl(A) = \mathfrak{F}\mathcal{F}int(1_{\mathfrak{F}} - A)$.

Definition 2.9. [24] *Let* (X, τ) *be an* $\mathfrak{F}\mathcal{F}ts$ *and* A *be an* $\mathfrak{F}\mathcal{F}s$. *Then* A *is said to be an Fermatean fuzzy (i) regular open set (*$\mathfrak{F}\mathcal{F}ros$ *in short) if* $A = \mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}cl(A))$. *(ii) regular closed set (*$\mathfrak{F}\mathcal{F}rcs$ *in short) if* $A = \mathfrak{F}\mathcal{F}cl(\mathfrak{F}\mathcal{F}int(A))$. *By Lemma 2.1, it follows that* A *is an* $\mathfrak{F}\mathcal{F}ros$ *iff* $\bar{A}$ *is an* $\mathfrak{F}\mathcal{F}rcs$.

Definition 2.10. [24] *Let* (X, τ) *be a* $\mathfrak{F}\mathcal{F}ts$. *Let* S *be a* $\mathfrak{F}\mathcal{F}s$ *of* X. *Then Fermatean*

(i) *fuzzy* δ *interior of* S *(briefly,* $\mathfrak{F}\mathcal{F}\delta int(S)$*) is defined by* $\mathfrak{F}\mathcal{F}\delta int(S) = \bigcup\{I : I \subseteq S \;\&\; I \text{ is a } \mathfrak{F}\mathcal{F}ro \text{ set in } X\}$.

(ii) fuzzy δ closure of S (briefly, $\mathfrak{F}\mathcal{F}\delta cl(S)$) is defined by $\mathfrak{F}\mathcal{F}\delta cl(S) = \bigcap\{A : S \subseteq A$ & A is a $\mathfrak{F}\mathcal{F}rc$ set in $X\}$.

Definition 2.11. [24] *Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Then a $\mathfrak{F}\mathcal{F}s$ S in X is said to be Fermatean*

(i) fuzzy δ-open (briefly, $\mathfrak{F}\mathcal{F}\delta o$) set if $S = \mathfrak{F}\mathcal{F}\delta int(S)$.

(ii) fuzzy $\delta\alpha$-open (briefly, $\mathfrak{F}\mathcal{F}\delta\alpha o$) set if $S \subseteq \mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}cl\ (\mathfrak{F}\mathcal{F}\delta int\ (S)))$.

(iii) fuzzy δ-semi open (briefly, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$) set if $S \subseteq \mathfrak{F}\mathcal{F}cl(\mathfrak{F}\mathcal{F}\delta int(S))$.

The complement of an $\mathfrak{F}\mathcal{F}\delta o$ (resp. $\mathfrak{F}\mathcal{F}\delta\alpha o$ & $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$) set is called a Fermatean fuzzy δ (resp. Fermatean fuzzy $\delta\alpha$ & Fermatean fuzzy δ-semi) closed (briefly, $\mathfrak{F}\mathcal{F}\delta c$ (resp. $\mathfrak{F}\mathcal{F}\delta\alpha c$ & $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$)) in X.

Definition 2.12. [24] *Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Let S be a $\mathfrak{F}\mathcal{F}s$ of X. Then Fermatean fuzzy*

(i) δ semi interior of S (briefly, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}int(S)$) is defined by $\mathfrak{F}\mathcal{F}\delta\mathcal{S}int(S) = \cup\{I : I \subseteq S$ & I is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set in $X\}$.

(ii) δ semi closure of S (briefly, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(S)$) is defined by $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(S) = \cap\{A : S \subseteq A$ & A is a $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ set in $X\}$.

Definition 2.13. [21, 22, 24] *A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be Fermatean fuzzy*

(i) continuous (briefly, $\mathfrak{F}\mathcal{F}Cts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}o$ set of X_1.

(ii) δ continuous (briefly, $\mathfrak{F}\mathcal{F}\delta Cts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\delta o$ set of X_1.

(iii) δ semi continuous (briefly, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set of X_1.

3. Fermatean fuzzy Z (resp. δ, $\delta\mathcal{S}$ and pre)-open and closed maps

In this section, we introduce Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) open maps and Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) closed maps in $\mathfrak{F}\mathcal{F}ts$ and obtain certain characterizations of these classes of maps.

Definition 3.1. *Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Then a $\mathfrak{F}\mathcal{F}s$ S in X is said to be Fermatean fuzzy pre open (briefly, $\mathfrak{F}\mathcal{F}\mathcal{P}o$) set if $S \subseteq \mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}cl(S))$.*

The complement of an $\mathfrak{F}\mathcal{F}\delta o$ (resp. $\mathfrak{F}\mathcal{F}\delta\alpha o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ & $\mathfrak{F}\mathcal{F}\mathcal{P}o$) set is called a Fermatean fuzzy δ (resp. Fermatean fuzzy $\delta\alpha$, Fermatean fuzzy δ-semi & Fermatean fuzzy pre) closed (briefly, $\mathfrak{F}\mathcal{F}\delta c$ (resp. $\mathfrak{F}\mathcal{F}\delta\alpha c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ & $\mathfrak{F}\mathcal{F}\mathcal{P}c$)) in X.

Definition 3.2. *Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Let S be a $\mathfrak{F}\mathcal{F}s$ of X. Then Fermatean fuzzy*

(i) pre interior of S (briefly, $\mathfrak{F}\mathcal{F}\mathcal{P}int(S)$) is defined by $\mathfrak{F}\mathcal{F}\mathcal{P}int(S) = \cup\{I : I \subseteq S \ \& \ I$ is a $\mathfrak{F}\mathcal{F}\mathcal{P}o$ set in $X\}$.

(ii) pre closure of S (briefly, $\mathfrak{F}\mathcal{F}\mathcal{P}cl(S)$) is defined by $\mathfrak{F}\mathcal{F}\mathcal{P}cl(S) = \cap\{A : S \subseteq A \ \& \ A$ is a $\mathfrak{F}\mathcal{F}\mathcal{P}c$ set in $X\}$.

Definition 3.3. *Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Then a $\mathfrak{F}\mathcal{F}s$ S in X is said to be a Fermatean fuzzy*

(i) Z-open (briefly, $\mathfrak{F}\mathcal{F}Zo$) set if $S \subseteq \mathfrak{F}\mathcal{F}cl(\mathfrak{F}\mathcal{F}\delta int(S)) \cap \mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}cl(S))$,

(ii) Z-closed (briefly, $\mathfrak{F}\mathcal{F}Zc$) set if $\mathfrak{F}\mathcal{F}int(\mathfrak{F}\mathcal{F}\delta cl(S)) \cap \mathfrak{F}\mathcal{F}cl(\mathfrak{F}\mathcal{F}int(S)) \subseteq S$.

The family of all $\mathfrak{F}\mathcal{F}Zo$ (resp. $\mathfrak{F}\mathcal{F}Zc$) sets of a space (X, τ) will be as always denoted by $\mathfrak{F}\mathcal{F}ZO(X)$ (resp. $\mathfrak{F}\mathcal{F}ZC(X)$).

Definition 3.4. *Let (X, τ) be a $\mathfrak{F}\mathcal{F}ts$. Let K be a $\mathfrak{F}\mathcal{F}s$ of X, then the Fermatean*

(i) fuzzy Z-interior of K is the union of all $\mathfrak{F}\mathcal{F}Zo$ sets contained in K and denoted by $\mathfrak{F}\mathcal{F}Zint(K)$.

(ii) fuzzy Z-closure of K is the intersection of all $\mathfrak{F}\mathcal{F}Zc$ sets containing K and denoted by $\mathfrak{F}\mathcal{F}Zcl(K)$.

Theorem 3.1. *Let K be a Fermatean fuzzy subset of a space (X, τ) Then*

(i) K is a $\mathfrak{F}\mathcal{F}Zo$ set iff $K = \mathfrak{F}\mathcal{F}Zint(K)$,

(ii) K is a $\mathfrak{F}\mathcal{F}Zc$ set iff $K = \mathfrak{F}\mathcal{F}Zcl(K)$.

Definition 3.5. *A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be Fermatean fuzzy*

(i) pre continuous (briefly, $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}\mathcal{P}o$ set of X_1.

(ii) Z continuous (briefly, $\mathfrak{F}\mathcal{F}ZCts$), if for each $\mathfrak{F}\mathcal{F}o$ set M of X_2, the set $h_{\mathfrak{F}}^{-1}(M)$ is $\mathfrak{F}\mathcal{F}Zo$ set of X_1.

Lemma 3.1. *Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a function. Then the following statements hold.*

(i) If A and B are Fermatean fuzzy subsets of X_1 such that $A \subseteq B$, then $h_{\mathfrak{F}}(A) \subseteq h_{\mathfrak{F}}(B)$.

(ii) If A and B are Fermatean fuzzy subsets of X_2 such that $A \subseteq B$, then $h_{\mathfrak{F}}^{-1}(A) \subseteq h_{\mathfrak{F}}^{-1}(B)$.

Lemma 3.2. *Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a function. If A is a Fermatean fuzzy subset of X_1 and B is a Fermatean fuzzy subset of X_2. Then*

(i) $h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(A)) \subseteq A$

(ii) $h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(A)) = A \Leftrightarrow h_{\mathfrak{F}}$ is surjective.

(iii) $h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(A)) \supseteq A$

(iv) $h_{\mathfrak{F}}^{-1}(h_{\mathfrak{F}}(A)) = A$ whenever $h_{\mathfrak{F}}$ is injective.

Theorem 3.2. *A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZCts$ iff the inverse image of every $\mathfrak{F}\mathcal{F}c$ set in X_2 is $\mathfrak{F}\mathcal{F}Zc$ in X_1.*

Definition 3.6. *Let (X_1, τ_1) and (X_2, τ_2) be two $\mathfrak{F}\mathcal{F}ts$. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) open map (briefly, $\mathfrak{F}\mathcal{F}O$ (resp. $\mathfrak{F}\mathcal{F}\delta O$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$, $\mathfrak{F}\mathcal{F}\mathcal{P}O$ and $\mathfrak{F}\mathcal{F}ZO$)) if the image of each $\mathfrak{F}\mathcal{F}o$ set in X_1 is $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$)-set in X_2.*

Definition 3.7. *Let (X_1, τ_1) and (X_2, τ_2) be two $\mathfrak{F}\mathcal{F}ts$. A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be Fermatean fuzzy (resp. δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) closed map (briefly, $\mathfrak{F}\mathcal{F}C$ (resp. $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$, $\mathfrak{F}\mathcal{F}\mathcal{P}C$ and $\mathfrak{F}\mathcal{F}ZC$)) if the image of each $\mathfrak{F}\mathcal{F}c$ set in X_1 is $\mathfrak{F}\mathcal{F}c$ (resp. $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$, $\mathfrak{F}\mathcal{F}\mathcal{P}c$ and $\mathfrak{F}\mathcal{F}Zc$)-set in X_2.*

Theorem 3.3. *Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a mapping. Then every*

(i) $\mathfrak{F}\mathcal{F}\delta O$ is $\mathfrak{F}\mathcal{F}O$.

(ii) $\mathfrak{F}\mathcal{F}O$ is $\mathfrak{F}\mathcal{F}\mathcal{P}O$.

(iii) $\mathfrak{F}\mathcal{F}\delta O$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$.

(iv) $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$ is $\mathfrak{F}\mathcal{F}ZO$.

(v) $\mathfrak{F}\mathcal{F}\mathcal{P}O$ is $\mathfrak{F}\mathcal{F}ZO$.

Proof. (i) Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be $\mathfrak{F}\mathcal{F}\delta O$ and L is a $\mathfrak{F}\mathcal{F}o$ set in X_1. Then $h_{\mathfrak{F}}(L)$ is $\mathfrak{F}\mathcal{F}\delta o$ set in X_2. Since every $\mathfrak{F}\mathcal{F}\delta os$ is $\mathfrak{F}\mathcal{F}os$, $h_{\mathfrak{F}}(L)$ is $\mathfrak{F}\mathcal{F}o$ set in X_2. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}O$.

(ii) Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be $\mathfrak{F}\mathcal{F}O$ and L is a $\mathfrak{F}\mathcal{F}o$ set in X_1. Then $h_{\mathfrak{F}}(L)$ is $\mathfrak{F}\mathcal{F}o$ set in X_2. Since every $\mathfrak{F}\mathcal{F}os$ is $\mathfrak{F}\mathcal{F}\mathcal{P}os$, $h_{\mathfrak{F}}(L)$ is $\mathfrak{F}\mathcal{F}\mathcal{P}o$ set in X_2. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\mathcal{P}O$.

(iii) Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be $\mathfrak{F}\mathcal{F}\delta O$ and L is a $\mathfrak{F}\mathcal{F}o$ set in X_1. Then $h_{\mathfrak{F}}(L)$ is $\mathfrak{F}\mathcal{F}\delta o$ set in X_2. Since every $\mathfrak{F}\mathcal{F}\delta os$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}os$, $h_{\mathfrak{F}}(L)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set in X_2. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$.

(iv) Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$ and L is a $\mathfrak{F}\mathcal{F}o$ set in X_1. Then $h_{\mathfrak{F}}(L)$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set in X_2. Since every $\mathfrak{F}\mathcal{F}\delta\mathcal{S}os$ is $\mathfrak{F}\mathcal{F}Zos$, $h_{\mathfrak{F}}(L)$ is $\mathfrak{F}\mathcal{F}Zo$ set in X_2. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZO$.

(iv) Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be $\mathfrak{F}\mathcal{F}\mathcal{P}O$ and L is a $\mathfrak{F}\mathcal{F}o$ set in X_1. Then $h_{\mathfrak{F}}(L)$ is $\mathfrak{F}\mathcal{F}\mathcal{P}o$ set in X_2. Since every $\mathfrak{F}\mathcal{F}\mathcal{P}os$ is $\mathfrak{F}\mathcal{F}Zos$, $h_{\mathfrak{F}}(L)$ is $\mathfrak{F}\mathcal{F}Zo$ set in X_2. Therefore $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZO$.

The converse of the Theorem 3.3 need not be true.

Example 3.1. Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1 and A_2 are defined as
$\mu_{A_1}(a) = 0.4,\ \lambda_{A_1}(a) = 0.1,$
$\mu_{A_1}(b) = 0.6,\ \lambda_{A_1}(b) = 0.3;$
$\mu_{A_2}(a) = 0.9,\ \lambda_{A_2}(a) = 0.2,$
$\mu_{A_2}(b) = 0.6,\ \lambda_{A_2}(b) = 0.3;$
Let $\tau_1 = \tau_2 = \tau = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2\}$ be a $\mathfrak{F}\mathcal{F}ts$ on X and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}O$ but not $\mathfrak{F}\mathcal{F}\delta O$. Since, A_2 is a $\mathfrak{F}\mathcal{F}o$ set in X_1 but $h_{\mathfrak{F}}(A_2) = A_2$ is not $\mathfrak{F}\mathcal{F}\delta o$ set in X_2.

Example 3.2. Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2 and A_3 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.8,$
$\mu_{A_1}(b) = 0.3,\ \lambda_{A_1}(b) = 0.7;$
$\mu_{A_2}(a) = 0.1,\ \lambda_{A_2}(a) = 0.9,$
$\mu_{A_2}(b) = 0.1,\ \lambda_{A_2}(b) = 0.9;$
$\mu_{A_3}(a) = 0.2,\ \lambda_{A_3}(a) = 0.8,$
$\mu_{A_3}(b) = 0.4,\ \lambda_{A_3}(b) = 0.6;$
Let $\tau_1 = \tau_2 = \tau = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3\}$ be a $\mathfrak{F}\mathcal{F}ts$ on X and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$ but not $\mathfrak{F}\mathcal{F}\delta O$. Since, A_1 is a $\mathfrak{F}\mathcal{F}o$ set in X_1 but $h_{\mathfrak{F}}(A_1) = A_1$ is not $\mathfrak{F}\mathcal{F}\delta o$ set in X_2.

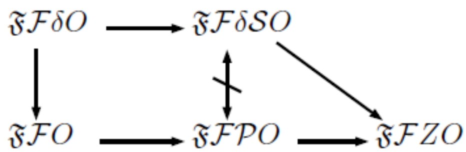
Example 3.3. Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, B_1 and B_2 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.7,$
$\mu_{A_1}(b) = 0.1,\ \lambda_{A_1}(b) = 0.8;$
$\mu_{A_2}(a) = 0.3,\ \lambda_{A_2}(a) = 0.6,$
$\mu_{A_2}(b) = 0.4,\ \lambda_{A_2}(b) = 0.5;$
$\mu_{B_1}(a) = 0.1,\ \lambda_{B_1}(a) = 0.9,$
$\mu_{B_1}(b) = 0.2,\ \lambda_{B_1}(b) = 0.9;$
$\mu_{B_2}(a) = 0.2,\ \lambda_{B_2}(a) = 0.3,$
$\mu_{B_2}(b) = 0.4,\ \lambda_{B_2}(b) = 0.7;$
Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2\}$ and $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ B_1,\ B_2\}$ are $\mathfrak{F}\mathcal{F}ts$'s on X and let $h_{\mathfrak{F}} : (X_2, \tau_2) \to (X_1, \tau_1)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}\mathcal{P}O$ but not $\mathfrak{F}\mathcal{F}O$. Since, B_2 is a $\mathfrak{F}\mathcal{F}o$ set in X_2 but $h_{\mathfrak{F}}(B_2) = B_2$ is not $\mathfrak{F}\mathcal{F}o$ set in X_1.

Example 3.4. Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, A_3, A_4, and A_5 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.8,$
$\mu_{A_1}(b) = 0.4,\ \lambda_{A_1}(b) = 0.6;$
$\mu_{A_2}(a) = 0.1,\ \lambda_{A_2}(a) = 0.9,$
$\mu_{A_2}(b) = 0.3,\ \lambda_{A_2}(b) = 0.7;$
$\mu_{A_3}(a) = 0.9,\ \lambda_{A_3}(a) = 0.1,$
$\mu_{A_3}(b) = 0.7,\ \lambda_{A_3}(b) = 0.3;$
$\mu_{A_4}(a) = 0.2,\ \lambda_{A_4}(a) = 0.8,$
$\mu_{A_4}(b) = 0.3,\ \lambda_{A_4}(b) = 0.7$. Let $\tau_1 = \tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3,\ A_4\}$ be $\mathfrak{F}\mathcal{F}ts$'s on X and let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZO$ but not $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$. Since, A_4 is $\mathfrak{F}\mathcal{F}o$ set in X_1 but $h_{\mathfrak{F}}(A_4) = A_4$ is not $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ set in X_2.

Example 3.5. Let $X_1 = X_2 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, A_3, A_4, B_1, B_2, B_3 and B_4 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.8,$
$\mu_{A_1}(b) = 0.4,\ \lambda_{A_1}(b) = 0.6;$
$\mu_{A_2}(a) = 0.1,\ \lambda_{A_2}(a) = 0.9,$
$\mu_{A_2}(b) = 0.3,\ \lambda_{A_2}(b) = 0.7;$
$\mu_{A_3}(a) = 0.9,\ \lambda_{A_3}(a) = 0.1,$
$\mu_{A_3}(b) = 0.7,\ \lambda_{A_3}(b) = 0.3;$
$\mu_{A_4}(a) = 0.2,\ \lambda_{A_4}(a) = 0.8,$
$\mu_{A_4}(b) = 0.3,\ \lambda_{A_4}(b) = 0.7;$
$\mu_{B_1}(a) = 0.4,\ \lambda_{B_1}(a) = 0.6,$
$\mu_{B_1}(b) = 0.5,\ \lambda_{B_1}(b) = 0.5;$

$\mu_{B_2}(a) = 0.6,\ \lambda_{B_2}(a) = 0.4,$
$\mu_{B_2}(b) = 0.6,\ \lambda_{B_2}(b) = 0.4;$
$\mu_{B_3}(a) = 0.7,\ \lambda_{B_3}(a) = 0.3,$
$\mu_{B_3}(b) = 0.6,\ \lambda_{B_3}(b) = 0.4;$
$\mu_{B_4}(a) = 0.4,\ \lambda_{B_4}(a) = 0.6,$
$\mu_{B_4}(b) = 0.4,\ \lambda_{B_4}(b) = 0.6;$
Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2,\ A_3,\ A_4\}$ and $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ B_1,\ B_2,\ B_3,\ B_4\}$ are $\mathfrak{F}\mathcal{F}ts$'s on X and let $h_{\mathfrak{F}} : (X_2, \tau_2) \to (X_1, \tau_1)$ be an identity function, Then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZO$ but not $\mathfrak{F}\mathcal{F}\mathcal{P}O$. Since, B_4 is a $\mathfrak{F}\mathcal{F}o$ set in X_2 but $h_{\mathfrak{F}}(B_4) = B_4$ is not $\mathfrak{F}\mathcal{F}\mathcal{P}o$ set in X_1.

Remark 3.1. *We obtain the following diagram from the results are discussed above.*



$\mathfrak{F}\mathcal{F}$ open mappings in $\mathfrak{F}\mathcal{F}ts$

Note: $K \to L$ denotes K implies L, but not conversely.

Theorem 3.4. *A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) mapping if and only if $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}(A)) \subseteq h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}cl(A))$ (resp. $\mathfrak{F}\mathcal{F}cl(h_{\mathfrak{F}}(A)) \subseteq h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}cl(A))$, $\mathfrak{F}\mathcal{F}\delta cl(h_{\mathfrak{F}}(A)) \subseteq h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}cl(A))$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}cl(h_{\mathfrak{F}}(A)) \subseteq h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}cl(A))$ and $\mathfrak{F}\mathcal{F}\mathcal{P}cl(h_{\mathfrak{F}}(A)) \subseteq h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}cl(A))$) for every Fermatean fuzzy set A of X_1.*

Proof. Suppose $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is a $\mathfrak{F}\mathcal{F}ZC$ function and A is any $\mathfrak{F}\mathcal{F}s$ in X_1. Then $\mathfrak{F}\mathcal{F}cl(A)$ is a $\mathfrak{F}\mathcal{F}c$ set in X_1. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZC$, $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}cl(A))$ is a $\mathfrak{F}\mathcal{F}Zc$ set in X_2. Then by Theorem 3.1 (ii), $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}cl(A))) = h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}cl(A))$. Therefore $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}(A)) \subseteq \mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}cl(A))) = h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}cl(A))$. Hence $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}(A)) \subseteq h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}cl\ (A))$.

Conversely, Let A be a $\mathfrak{F}\mathcal{F}c$ set in X_1. Then $\mathfrak{F}\mathcal{F}cl(A) = A$ and so $h_{\mathfrak{F}}(A) = h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}cl(A))$. By our assumption $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}(A)) \subseteq h_{\mathfrak{F}}(A)$. But $h_{\mathfrak{F}}(A) \subseteq \mathfrak{F}\mathcal{F}Zcl\ (h_{\mathfrak{F}}(A))$. Hence $\mathfrak{F}\mathcal{F}Zcl(h_{\mathfrak{F}}(A)) = h_{\mathfrak{F}}(A)$ and therefore by Theorem 3.1 (ii), $h_{\mathfrak{F}}(A)$ is $\mathfrak{F}\mathcal{F}Zcs$ in X_2. Thus $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}ZC$ map.

Other cases are similar.

Theorem 3.5. *A map $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$,*

$\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) mapping if and only if for each Fermatean fuzzy set S of X_2 and for each $\mathfrak{F}\mathcal{F}o$ set U of X_1 containing $h_{\mathfrak{F}}^{-1}(S)$ there exists a $\mathfrak{F}\mathcal{F}Zo$ (resp. $\mathfrak{F}\mathcal{F}o$, $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ and $\mathfrak{F}\mathcal{F}\mathcal{P}o$) set V of X_2 such that $S \subseteq V$ and $h_{\mathfrak{F}}^{-1}(V) \subseteq U$.

Proof. Suppose $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}ZC$ map. Let S be any Fermatean fuzzy set in X_2 and U be a $\mathfrak{F}\mathcal{F}Zo$ set of X_1 such that $h_{\mathfrak{F}}^{-1}(S) \subseteq U$. Then $V = (h_{\mathfrak{F}}(U^c))^c$ is $\mathfrak{F}\mathcal{F}Zo$ set containing S such that $h_{\mathfrak{F}}^{-1}(V) \subseteq U$. Conversely, Let S be a $\mathfrak{F}\mathcal{F}c$ set of X_1. Then $h_{\mathfrak{F}}^{-1}((h_{\mathfrak{F}}(S))^c) \subseteq S^c$ and S^c is $\mathfrak{F}\mathcal{F}os$ in X_1.

By assumption, there exists a $\mathfrak{F}\mathcal{F}Zo$ set V of X_2 such that $(h_{\mathfrak{F}}(S))^c \subseteq V$ and $h_{\mathfrak{F}}^{-1}(V) \subseteq S^c$ and so $S \subseteq (h_{\mathfrak{F}}^{-1}(V))^c$. Hence $V^c \subseteq h_{\mathfrak{F}}(S) \subseteq h_{\mathfrak{F}}((h_{\mathfrak{F}}^{-1}(V))^c) \subseteq V^c$, which implies $h_{\mathfrak{F}}(S) = V^c$. Since V^c is $\mathfrak{F}\mathcal{F}Zcs$, $h_{\mathfrak{F}}(S)$ is $\mathfrak{F}\mathcal{F}Zcs$ and $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZC$ map.

Other cases are similar.

Remark 3.2. *The composition of two $\mathfrak{F}\mathcal{F}ZO$ (resp. $\mathfrak{F}\mathcal{F}O$, $\mathfrak{F}\mathcal{F}\delta O$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$ and $\mathfrak{F}\mathcal{F}\mathcal{P}O$) maps need not be a $\mathfrak{F}\mathcal{F}ZO$ (resp. $\mathfrak{F}\mathcal{F}O$, $\mathfrak{F}\mathcal{F}\delta O$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$ and $\mathfrak{F}\mathcal{F}\mathcal{P}O$) map, which is shown in the following example.*

Example 3.6. Let $X_1 = X_2 = X_3 = X = \{a, b\}$ and the $\mathfrak{F}\mathcal{F}s$'s A_1, A_2, B_1, B_2 and C_1 are defined as
$\mu_{A_1}(a) = 0.2,\ \lambda_{A_1}(a) = 0.7,$
$\mu_{A_1}(b) = 0.1,\ \lambda_{A_1}(b) = 0.8;$
$\mu_{A_2}(a) = 0.3,\ \lambda_{A_2}(a) = 0.6,$
$\mu_{A_2}(b) = 0.4,\ \lambda_{A_2}(b) = 0.5;$
$\mu_{B_1}(a) = 0.1,\ \lambda_{B_1}(a) = 0.9,$
$\mu_{B_1}(b) = 0.2,\ \lambda_{B_1}(b) = 0.9;$
$\mu_{B_2}(a) = 0.2,\ \lambda_{B_2}(a) = 0.3,$
$\mu_{B_2}(b) = 0.4,\ \lambda_{B_2}(b) = 0.7;$
$\mu_{C_1}(a) = 0.3,\ \lambda_{C_1}(a) = 0.2,$
$\mu_{C_1}(b) = 0.7,\ \lambda_{C_1}(b) = 0.4;$
Let $\tau_1 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ A_1,\ A_2\}$, $\tau_2 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ B_1,\ B_2\}$ and $\tau_3 = \{0_{\mathfrak{F}},\ 1_{\mathfrak{F}},\ C_1\}$ are $\mathfrak{F}\mathcal{F}ts$'s on X and

let $h_{\mathfrak{F}} : (X_3, \tau_3) \to (X_2, \tau_2)$ and $g_{\mathfrak{F}} : (X_2, \tau_2) \to (X_1, \tau_1)$ be an identity function, Then $h_{\mathfrak{F}}$ and $g_{\mathfrak{F}}$ are $\mathfrak{F}\mathcal{F}ZO$ but $(g_{\mathfrak{F}} \circ h_{\mathfrak{F}})$ is not $\mathfrak{F}\mathcal{F}ZO$. Since, C_1 is a $\mathfrak{F}\mathcal{F}o$ set in X_3 but $(g_{\mathfrak{F}} \circ h_{\mathfrak{F}})(C_1) = C_1$ is not $\mathfrak{F}\mathcal{F}Zo$ set in X_1.

Theorem 3.6. *Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a $\mathfrak{F}\mathcal{F}C$ map and $g_{\mathfrak{F}} : (X_2, \tau_2) \to (X_3, \tau_3)$ be a $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map. Then their composition $g_{\mathfrak{F}} \circ h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_3, \tau_3)$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$).*

Proof. Let F be a $\mathfrak{F}\mathcal{F}c$ set in X_1. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}C$, $h_{\mathfrak{F}}(F)$ is $\mathfrak{F}\mathcal{F}cs$ in X_2. Since

$g_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$), $g_{\mathfrak{F}}(h_{\mathfrak{F}}(F)) = (g_{\mathfrak{F}} \circ h_{\mathfrak{F}})(F)$ is $\mathfrak{F}\mathcal{F}Zc$ (resp. $\mathfrak{F}\mathcal{F}c$, $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ and $\mathfrak{F}\mathcal{F}\mathcal{P}c$)-set in X_3. Hence $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map.

Theorem 3.7. *Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ and $g_{\mathfrak{F}} : (X_2, \tau_2) \to (X_3, \tau_3)$ be two mappings such that their composition $g_{\mathfrak{F}} \circ h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_3, \tau_3)$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map. Then the followings are true.*

(i) If $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Cts$ and surjective, then $g_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map.

(ii) If $g_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZIrr$ (resp. $\mathfrak{F}\mathcal{F}Irr$, $\mathfrak{F}\mathcal{F}\delta Irr$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Irr$ and $\mathfrak{F}\mathcal{F}\mathcal{P}Irr$) and injective, then $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map.

Proof. (i) Let A be a $\mathfrak{F}\mathcal{F}c$ set of X_2. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}Cts$ map, $h_{\mathfrak{F}}^{-1}(A)$ is $\mathfrak{F}\mathcal{F}c$ in X_1. Since $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map, $(g_{\mathfrak{F}} \circ h_{\mathfrak{F}})(h_{\mathfrak{F}}^{-1}(A))$ is $\mathfrak{F}\mathcal{F}Zc$ (resp. $\mathfrak{F}\mathcal{F}c$, $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ and $\mathfrak{F}\mathcal{F}\mathcal{P}c$) set in Z. Since $h_{\mathfrak{F}}$ is surjective, $g_{\mathfrak{F}}(A)$ is $\mathfrak{F}\mathcal{F}Zc$ (resp. $\mathfrak{F}\mathcal{F}c$, $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ and $\mathfrak{F}\mathcal{F}\mathcal{P}c$) set in X_3. Hence $g_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map.

(ii) Let B be any $\mathfrak{F}\mathcal{F}c$ set of X_1. Since $g_{\mathfrak{F}} \circ h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map, $(g_{\mathfrak{F}} \circ h_{\mathfrak{F}})(B)$ is $\mathfrak{F}\mathcal{F}Zc$ (resp. $\mathfrak{F}\mathcal{F}c$, $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ and $\mathfrak{F}\mathcal{F}\mathcal{P}c$)-set in X_3. Since $g_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZIrr$ (resp. $\mathfrak{F}\mathcal{F}Irr$, $\mathfrak{F}\mathcal{F}\delta Irr$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Irr$ and $\mathfrak{F}\mathcal{F}\mathcal{P}Irr$), $g_{\mathfrak{F}}^{-1}(g_{\mathfrak{F}} \circ h_{\mathfrak{F}}(B))$ is $\mathfrak{F}\mathcal{F}Zc$ (resp. $\mathfrak{F}\mathcal{F}c$, $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ and $\mathfrak{F}\mathcal{F}\mathcal{P}c$)-set in X_2. Since $g_{\mathfrak{F}}$ is injective, $h_{\mathfrak{F}}(B)$ is $\mathfrak{F}\mathcal{F}Zc$ (resp. $\mathfrak{F}\mathcal{F}c$, $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ and $\mathfrak{F}\mathcal{F}\mathcal{P}c$)-set in X_2. Hence $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map.

Theorem 3.8. *Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map.*

(i) If A is $\mathfrak{F}\mathcal{F}c$ set of X_1, then the restriction $h_{\mathfrak{F}_A} : (X_{1_A}, \tau_{1_A}) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map.

(ii) If $A = h_{\mathfrak{F}}^{-1}(B)$ for some $\mathfrak{F}\mathcal{F}c$ set B of X_2, then the restriction $h_{\mathfrak{F}_A} : (X_{1_A}, \tau_{1_A}) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map.

Proof. (i) Let B be any $\mathfrak{F}\mathcal{F}c$ set of A. Then $B = A \cap L$ for some $\mathfrak{F}\mathcal{F}c$ set L of X_1 and so B is $\mathfrak{F}\mathcal{F}cs$ in X_1. By hypothesis, $h_{\mathfrak{F}}(B)$ is $\mathfrak{F}\mathcal{F}Zc$ (resp. $\mathfrak{F}\mathcal{F}c$, $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ and $\mathfrak{F}\mathcal{F}\mathcal{P}c$)-set in X_2. But $h_{\mathfrak{F}}(B) = h_{\mathfrak{F}_A}(B)$, therefore $h_{\mathfrak{F}_A}$ is a $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map.

(ii) Let D be a $\mathfrak{F}\mathcal{F}c$ set of A. Then $D = A \cap H$, for some $\mathfrak{F}\mathcal{F}c$ set H in X_1. Now, $h_{\mathfrak{F}_A}(D) = h_{\mathfrak{F}}(D) = h_{\mathfrak{F}}(A \cap H) = h_{\mathfrak{F}}(h_{\mathfrak{F}}^{-1}(B) \cap H) = B \cap h_{\mathfrak{F}}(H)$. Since $h_{\mathfrak{F}}$

is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$), $h_{\mathfrak{F}}(H)$ is $\mathfrak{F}\mathcal{F}Zc$ (resp. $\mathfrak{F}\mathcal{F}c$, $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ and $\mathfrak{F}\mathcal{F}\mathcal{P}c$)-set in X_2. Hence $h_{\mathfrak{F}_A}$ is a $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map.

Theorem 3.9. *A function $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is $\mathfrak{F}\mathcal{F}ZO$ (resp. $\mathfrak{F}\mathcal{F}O$, $\mathfrak{F}\mathcal{F}\delta O$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$ and $\mathfrak{F}\mathcal{F}\mathcal{P}O$) map if and only if $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A)) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(A))$ (resp. $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A)) \subseteq \mathfrak{F}\mathcal{F}int(h_{\mathfrak{F}}(A))$, $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A)) \subseteq \mathfrak{F}\mathcal{F}\delta int(h_{\mathfrak{F}}(A))$, $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}$ $int(A)) \subseteq \mathfrak{F}\mathcal{F}\delta\mathcal{S}int(h_{\mathfrak{F}}(A))$ and $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A)) \subseteq \mathfrak{F}\mathcal{F}\mathcal{P}int(h_{\mathfrak{F}}(A))$), for every Fermatean fuzzy set A of X_1.*

Proof. Suppose $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ is a $\mathfrak{F}\mathcal{F}ZO$ function and A is $\mathfrak{F}\mathcal{F}s$ in X_1. Then $\mathfrak{F}\mathcal{F}int(A)$ is a $\mathfrak{F}\mathcal{F}o$ set in X_1. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZO$, $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A))$ is a $\mathfrak{F}\mathcal{F}Zo$ set. Since $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A))) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(A))$, $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A)) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(A))$.

Conversely, $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A)) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(A))$ for every Fermatean fuzzy set A in X_1. Let U be a $\mathfrak{F}\mathcal{F}o$ set in X_1. Then $\mathfrak{F}\mathcal{F}int(U) = U$ and by hypothesis, $h_{\mathfrak{F}}(U) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(U))$. But $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(U)) \subseteq h_{\mathfrak{F}}(U)$. Therefore, $h_{\mathfrak{F}}(U) = \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(U))$. Then by Theorem 3.1 (i), $h_{\mathfrak{F}}(U)$ is $\mathfrak{F}\mathcal{F}Zos$. Hence $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}ZO$ map.

Other cases are similar.

Definition 3.8. *A Fermatean fuzzy set A in a $\mathfrak{F}\mathcal{F}ts$ (X, τ) is called a Fermatean fuzzy (δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) q-neighborhood of a Fermatean fuzzy point x_r if there exists a $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) set V in (X, τ) such that $x_r qV \subseteq A$.*

Definition 3.9. *Let A and B be any two $\mathfrak{F}\mathcal{F}s$'s of a $\mathfrak{F}\mathcal{F}ts$'s. Then A is Fermatean fuzzy (δ, $\delta\mathcal{S}$, $\mathcal{P}$ and Z) q-neighbourhood (briefly, $\mathfrak{F}\mathcal{F}q$-nbhd (resp. $\mathfrak{F}\mathcal{F}\delta q$-nbhd, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}q$-nbhd, $\mathfrak{F}\mathcal{F}\mathcal{P}q$-nbhd and $\mathfrak{F}\mathcal{F}Zq$-nbhd)) with B if there exists a $\mathfrak{F}\mathcal{F}o$ (resp. $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$, $\mathfrak{F}\mathcal{F}\mathcal{P}o$ and $\mathfrak{F}\mathcal{F}Zo$) set O with $AqO \subseteq B$.*

Theorem 3.10. *Let $h_{\mathfrak{F}} : (X_1, \tau_1) \to (X_2, \tau_2)$ be a mapping. Then the following statements are equivalent.*

(i) $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}ZO$ (resp. $\mathfrak{F}\mathcal{F}O$, $\mathfrak{F}\mathcal{F}\delta O$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$ and $\mathfrak{F}\mathcal{F}\mathcal{P}O$) mapping,

(ii) For a subset A of X_1, $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A)) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(A))$ (resp. $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int$ $(A)) \subseteq \mathfrak{F}\mathcal{F}int(h_{\mathfrak{F}}(A))$, $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}$ $int(A)) \subseteq \mathfrak{F}\mathcal{F}\delta int(h_{\mathfrak{F}}(A))$, $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A)) \subseteq \mathfrak{F}\mathcal{F}\delta\mathcal{S}int(h_{\mathfrak{F}}(A))$ and $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A)) \subseteq \mathfrak{F}\mathcal{F}\mathcal{P}int(h_{\mathfrak{F}}(A))$.

(iii) For each $x_\alpha \in X_1$ and for each $\mathfrak{F}\mathcal{F}q$-nbhd U of x_α in X_1, there exists A $\mathfrak{F}\mathcal{F}Zq$-nbhd (resp. $\mathfrak{F}\mathcal{F}q$-nbhd, $\mathfrak{F}\mathcal{F}\delta q$-nbhd, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}q$-nbhd and $\mathfrak{F}\mathcal{F}\mathcal{P}q$-nbhd) W of $h_{\mathfrak{F}}(x_\alpha)$ in X_2 such that $W \subseteq h_{\mathfrak{F}}(U)$.

Proof. (i) $\Rightarrow$ (ii): Suppose $h_{\mathfrak{F}} : (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ is a $\mathfrak{F}\mathcal{F}ZO$ function and $A \subseteq X_1$. Then $\mathfrak{F}\mathcal{F}int(A)$ is a $\mathfrak{F}\mathcal{F}o$ set in X_1. Since $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZO$ map, $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A))$ is a $\mathfrak{F}\mathcal{F}Zo$ set. Since $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A))) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(A))$, $h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(A)) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(A))$. This proves (ii).

(ii) $\Rightarrow$ (iii): Let $x_\alpha \in X_1$ and U be any arbitrary $\mathfrak{F}\mathcal{F}$*q-nbhd* of x_α in X_1. Then there exists a $\mathfrak{F}\mathcal{F}o$ set G such that $x_\alpha \in G \subseteq U$. By (ii), $h_{\mathfrak{F}}(G) = h_{\mathfrak{F}}(\mathfrak{F}\mathcal{F}int(G)) \subseteq \mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(G))$. But, $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(G)) \subseteq h_{\mathfrak{F}}(G)$. Therefore, $\mathfrak{F}\mathcal{F}Zint(h_{\mathfrak{F}}(G)) = h_{\mathfrak{F}}(G)$ and hence $h_{\mathfrak{F}}(G)$ is $\mathfrak{F}\mathcal{F}Zo$ in X_2. Since $x_\alpha \in G \subseteq U$, $h_{\mathfrak{F}}(x_\alpha) \in h_{\mathfrak{F}}(G) \subseteq h_{\mathfrak{F}}(U)$ and so (iii) holds, by taking $W = h_{\mathfrak{F}}(G)$.

(iii) $\Rightarrow$ (i): Let U be any $\mathfrak{F}\mathcal{F}o$ set in X_1. Let $x_\alpha \in U$ and $h_{\mathfrak{F}}(x_\alpha) = y_\beta$. Then for each $x_\alpha \in U$, $y_\beta \in h_{\mathfrak{F}}(U)$, by assumption there exists a $\mathfrak{F}\mathcal{F}$*qZ-nbhd* $W(y_\beta)$ of y_β in X_2 such that $W(y_\beta) \subseteq h_{\mathfrak{F}}(U)$. Since $W(y_\beta)$ is a $\mathfrak{F}\mathcal{F}$*qZ-nbhd* of y_β, there exists a $\mathfrak{F}\mathcal{F}Zo$ set $V(y_\beta)$ in X_2 such that $y_\beta \in V(y_\beta) \subseteq W(y_\beta)$. Therefore, $h_{\mathfrak{F}}(U) = \cup\{V(y_\beta) | y_\beta \in h_{\mathfrak{F}}(U)\}$. Since the union of $\mathfrak{F}\mathcal{F}Zo$ sets is $\mathfrak{F}\mathcal{F}Zo$, $h_{\mathfrak{F}}(U)$ is a $\mathfrak{F}\mathcal{F}Zo$ set in X_2. Thus, $h_{\mathfrak{F}}$ is a $\mathfrak{F}\mathcal{F}ZO$ map.

Other cases are similar.

Theorem 3.11. *For any bijective map $h_{\mathfrak{F}} : (X_1, \tau_1) \rightarrow (X_2, \tau_2)$ the following statements are equivalent:*

(i) $h_{\mathfrak{F}}^{-1} : (X_2, \tau_2) \rightarrow (X_1, \tau_1)$ is $\mathfrak{F}\mathcal{F}ZCts$ (resp. $\mathfrak{F}\mathcal{F}Cts$, $\mathfrak{F}\mathcal{F}\delta Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$ and $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$).

(ii) $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZO$ (resp. $\mathfrak{F}\mathcal{F}O$, $\mathfrak{F}\mathcal{F}\delta O$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$ and $\mathfrak{F}\mathcal{F}\mathcal{P}O$) map.

(iii) $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map.

Proof. (i) $\rightarrow$ (ii): Let U be a $\mathfrak{F}\mathcal{F}o$ set in X_1. By assumption, $(h_{\mathfrak{F}}^{-1})^{-1}(U) = h_{\mathfrak{F}}(U)$ is $\mathfrak{F}\mathcal{F}Zo$ (resp. $\mathfrak{F}\mathcal{F}o$, $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ and $\mathfrak{F}\mathcal{F}\mathcal{P}o$)-set in X_2 and so $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZO$ (resp. $\mathfrak{F}\mathcal{F}O$, $\mathfrak{F}\mathcal{F}\delta O$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}O$ and $\mathfrak{F}\mathcal{F}\mathcal{P}O$) map.

(ii) $\rightarrow$ (iii): Let F be a $\mathfrak{F}\mathcal{F}c$ set of X_1. Then F^c is a $\mathfrak{F}\mathcal{F}o$ set in X_1. By assumption $h_{\mathfrak{F}}(F^c)$ is $\mathfrak{F}\mathcal{F}Zo$ (resp. $\mathfrak{F}\mathcal{F}o$, $\mathfrak{F}\mathcal{F}\delta o$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}o$ and $\mathfrak{F}\mathcal{F}\mathcal{P}o$) set in X_2. But $h_{\mathfrak{F}}(F^c) = (h_{\mathfrak{F}}(F))^c$. Therefore $h_{\mathfrak{F}}(F)$ is $\mathfrak{F}\mathcal{F}Zc$ (resp. $\mathfrak{F}\mathcal{F}c$, $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ and $\mathfrak{F}\mathcal{F}\mathcal{P}c$) set in X_2. Hence, $h_{\mathfrak{F}}$ is $\mathfrak{F}\mathcal{F}ZC$ (resp. $\mathfrak{F}\mathcal{F}C$, $\mathfrak{F}\mathcal{F}\delta C$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}C$ and $\mathfrak{F}\mathcal{F}\mathcal{P}C$) map.

(iii) $\Rightarrow$ (i): Let F be a $\mathfrak{F}\mathcal{F}Zc$ (resp. $\mathfrak{F}\mathcal{F}c$, $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ and $\mathfrak{F}\mathcal{F}\mathcal{P}c$) set of X_1. By assumption, $h_{\mathfrak{F}}(F)$ is $\mathfrak{F}\mathcal{F}Zc$ (resp. $\mathfrak{F}\mathcal{F}c$, $\mathfrak{F}\mathcal{F}\delta c$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}c$ and $\mathfrak{F}\mathcal{F}\mathcal{P}c$) set in X_2. But $h_{\mathfrak{F}}(F) = (h_{\mathfrak{F}}^{-1})^{-1}(F)$ and therefore by Theorem 3.2, $h_{\mathfrak{F}}^{-1}$ is $\mathfrak{F}\mathcal{F}ZCts$ (resp. $\mathfrak{F}\mathcal{F}Cts$, $\mathfrak{F}\mathcal{F}\delta Cts$, $\mathfrak{F}\mathcal{F}\delta\mathcal{S}Cts$ and $\mathfrak{F}\mathcal{F}\mathcal{P}Cts$).

4. Application

Entropy as a measure of fuzziness was first proposed by Zadeh [31]. Later many mathematicians defined several entropy measures. In this section, we focus on defining an entropy measure for $\mathfrak{F}\mathcal{F}s$ that connects the degree of membership and non-membership. As an example, we have applied the proposed entropy measure in decision making.

Definition 4.1. *Let $A = \{< x, \mu_A(x), \lambda_A(x) | x \in X\}$ be a $\mathfrak{F}\mathcal{F}s$ in X. The new entropy measure for A denoted by $\varepsilon_{\mathfrak{F}\mathcal{F}s}(A)$, is a function, $\varepsilon_{\mathfrak{F}\mathcal{F}s} : \tau_{\mathfrak{F}\mathcal{F}s}(X) \to [0, 1]$ and is defined as $\varepsilon_{\mathfrak{F}\mathcal{F}s}(A) = 1 - \frac{1}{n}\sum_{i=1}^{n}(\mu_A - \lambda_A)^2$; for every'$x_i \in A$, where $\tau_{\mathfrak{F}\mathcal{F}s}(X)$ denote the family of all $\mathfrak{F}\mathcal{F}s$'s on X.*

Example 4.1. A global automaker group decided to start their production unit in India. The following factors are considered as the parameters:

e_1: Good site characteristics

e_2: Availability of utilities

e_3: Lower disaster risk

e_4: Good business climate

e_5: Proximity to original equipment makers

After a deep searching process, four places P_1, P_2, P_3, and P_4 are shortlisted to start their production unit. Three decision-makers are selected to take a better decision. After a careful examination of each of the parameters e_j, $j = 1, 2, \ldots, 5$, the decision-makers assigned their subjective weights, which are given in Table 1.

Table 1. Reviews Based on the Decision Makers

	$e1$	$e2$	$e3$	$e4$	$e5$
$P1$	$< P1, e1; 0.32, 0.62 >$	$< P1, e2; 0.50, 0.49 >$	$< P1, e3; 0.55, 0.10 >$	$< P1, e4; 0.60, 0.10 >$	$< P1, e5; 0.50, 0.02 >$
$P2$	$< P2, e1; 0.62, 0.25 >$	$< P2, e2; 0.09, 0.60 >$	$< P2, e3; 0.22, 0.47 >$	$< P2, e4; 0.30, 0.18 >$	$< P2, e5; 0.30, 0.18 >$
$P3$	$< P3, e1; 0.32, 0.17 >$	$< P3, e2; 0.10, 0.3 >$	$< P3, e3; 0.88, 0.10 >$	$< P1, e4; 0.10, 0.42 >$	$< P1, e5; 0.10, 0.42 >$
$P4$	$< P4, e1; 0.20, 0.55 >$	$< P4, e2; 0.15, 0.25 >$	$< P4, e3; 0.02, 0.45 >$	$< P4, e4; 0.42, 0.02 >$	$< P4, e5; 0.60, 0.08 >$

Clearly, all values in the Table 1 are $\mathfrak{F}\mathcal{F}s$'s. Now we calculate the $\varepsilon_{\mathfrak{F}\mathcal{F}s}$ of each Parameters.

Table 2. Entropy measure of each Parameter based on the review of decision makers.

Table 3. Entropy measure of each Places.

From Table 3, it is clear that

$$\varepsilon_{\mathfrak{F}\mathcal{F}s}(P_4) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(P_2) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(P_1) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(P_3).$$

	$e1$	$e2$	$e3$	$e4$	$e5$
$P1$	0.91	1	0.80	**0.75**	0.77
$P2$	0.86	**0.74**	0.94	0.99	0.99
$P3$	0.98	0.95	**0.39**	0.90	0.90
$P4$	0.88	0.99	0.82	0.84	**0.73**

	$\varepsilon_{\mathfrak{F}\mathcal{F}s}(Pi)$
$P1$	4.23
$P2$	4.51
$P3$	**4.11**
$P4$	4.25

Thus, we can rank the places as

$$\varepsilon_{\mathfrak{F}\mathcal{F}s}(P_4) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(P_2) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(P_1) < \varepsilon_{\mathfrak{F}\mathcal{F}s}(P_3).$$

This implies that P_4 is the optimal solution. Hence, the automaker group can choose place P_4 to start their production unit in India.

5. Conclusion

In this paper, we investigated Fermatean fuzzy Z-open maps and Fermatean fuzzy Z-closed maps in the setting of Fermatean fuzzy topological spaces. The main objective of the study was to develop and analyze these new classes of mappings and to examine their basic properties and relationships. Our results show that these concepts provide a useful framework for studying the behavior of mappings in Fermatean fuzzy topological spaces and contribute to the broader development of Fermatean fuzzy topology.

In addition, we introduced an entropy measure for Fermatean fuzzy sets and demonstrated its practical usefulness through an illustrative application. The numerical results obtained from the example show that the proposed entropy measure effectively quantifies uncertainty and can be applied in decision-making problems involving imprecise information. These findings highlight the significance of the proposed approach and indicate that it may be useful in other areas of generalized fuzzy systems as well.

Future research may focus on Fermatean fuzzy Z-homeomorphism functions and related mapping concepts, which may lead to deeper insights into topological equivalence in the Fermatean fuzzy setting.

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